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EXACT SOLUTION OF WAVE SCATTERING BY IRREGULAR SURFACES.(U)

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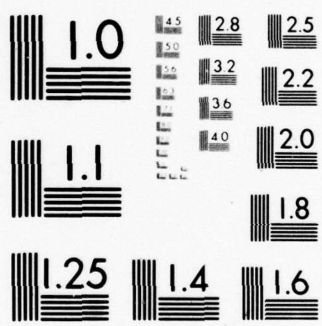

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U. S. Navy Underwater Sound Laboratory
Fort Trumbull, New London, Connecticut

EXACT SOLUTION OF WAVE SCATTERING BY IRREGULAR SURFACES.

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H. W. Marsh

USL Technical Memorandum No. 1100-22-56

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ABSTRACT

The problem which Lord Rayleigh considered, (Reflection from a Corrugated Surface) and which several have treated approximately in recent years, is developed in a general form. An exact solution is obtained by extensive use of Wiener's Generalized Harmonic Analysis. This solution is readily extended to include electromagnetic waves, general elastic waves, and non-planar, non-harmonic sources. Numerical evaluation of the formulae is considered by means of an operational representation, from which various types of successive approximations can be constructed.

Applications can be of special interest in scatter communications and in underwater sound transmission and reverberation. Experimental evidence from the latter area indicates that a rather detailed theory is necessary for an adequate account of the situation.

INTRODUCTION

The scattering of waves by an irregular surface was of interest initially in connection with diffraction gratings. Here an application of Huygen's principle was adequate to account for the qualitative features of the various orders of the diffracted spectra. Subsequently, Rayleigh considered the reflection of waves of normal incidence by a simply (harmonic) corrugated surface, and showed that the amplitude of the several spectral orders could be obtained by inverting an infinite matrix. More recently, several writers have obtained results of a statistical nature.

It is appropriate to discuss the problem at this International Congress, considering that important earlier contributions have been

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) made by Rayleigh, an Englishman, Eckart, an American, and Brekhovskikh, a Russian, among a number of others.

The material presented here is the first part of what will be a three part treatment. In this first part, for reasons that will be explained, we confine our attention to a rather special but fundamental case. We shall then anticipate breifly parts two and three, which respectively generalize and apply the results.

STATEMENT OF THE PROBLEM

We wish to consider the scattering of an arbitrary wave incident upon the irregular surface separating two media of different propagation constants and impedances. However, it results that the most general theory can be made to depend simply on a much more modest yet fundamental one. This fundamental theory will be left in an heuristic form, in order to avoid presenting the exceedingly heavy analysis required for a rigorous treatment.

GEOMETRY OF ONE DIMENSIONAL SURFACE

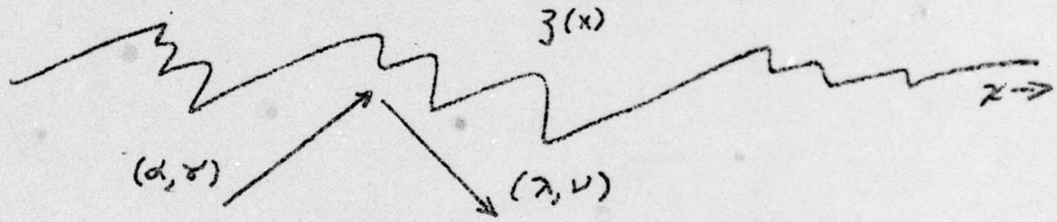

Fig. 1

Thus, we first consider the case of a plane, simple harmonic scalar (acoustic) wave, incident upon an irregular one dimensional surface $z(x)$[1] (as in Figure 1) on which the wave potential vanishes. Make the following identifications:

---

[1] The one dimensional surface is an ordinary one whose generators form a parallel family (in this case, parallel to the y axis). The incident wave is parallel to these generators, and hence also the scattered wave.

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ω = angular frequency of incident wave

k = wave number of incident wave

α, γ = direction cosines of incident wave normal with respect to x, z, respectively

g = scattered wave

application of the boundary condition of the free surface $z(x)$ gives the equation

$$g\{x, z(x), t\} + e^{i\omega t - ik(\alpha x + \gamma z(x))} = 0 \qquad (1)$$

It is convenient to use non-dimensional coordinates, and to suppress the superfluous time factor. Further, let h denote the root-mean-square height of the surface, i.e.,

$$h^2 = \langle z^2(x) \rangle$$

and let

$$kx = \xi \; ; \; kz(x) = \sigma \zeta(\xi)$$

We seek a plane wave representation for g by writing

$$g e^{-i\omega t} = \int e^{-i(\lambda\xi - \nu k z)} dG(\lambda) \qquad (2)$$

in which

$$\lambda^2 + \nu^2 = 1$$

$$0 \leq \arg \nu < \pi$$

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$G(\lambda)$ denotes the generalized spectrum of $g(z)$ and the convention on ν assures that the elementary scattered waves are outgoing.

Collecting results, we take as our point of departure the equation

$$-e^{-\iota(\alpha\xi+\gamma\sigma\zeta)} = \int e^{-\iota(\lambda\xi-\nu\sigma\zeta)}\, dG(\lambda) \qquad (3)$$

for

$$-\infty < \xi < \infty$$

THE ROLE OF GENERALIZED HARMONIC ANALYSIS IN THE PROBLEM.

Our philosophy in studying Eq. (3) is as follows: $G(\lambda)$, the generalized spectrum in wave - number space of the scattered wave, is according to Weiner, related to that wave by

$$e^{\iota\omega t}G(\lambda) = \mathrm{l.i.m.}\ \frac{1}{2\pi}\left[\int_{-A}^{-1} + \int_{1}^{A}\right] \frac{g(\xi)e^{\iota\lambda\xi}}{\iota\xi}\, d\xi$$

$$+ \frac{1}{2\pi}\int_{-1}^{1} g(\xi)\frac{[1-e^{\iota\lambda\xi}]}{\iota\xi}\, d\xi \qquad (4)$$

$$\equiv W_\lambda\{g(\xi)\}$$

W_λ is the symbolic operator representing this generalized spectrum. Corresponding to $G(\lambda)$ there is a correlation function $\varphi(\xi)$ possessing the usual Fourier mate

$$\Lambda(\lambda) = \frac{1}{2\pi}\int e^{i\lambda\xi}\varphi(\xi)\,d\xi \tag{5}$$

The physical meaning of this transform is this: $\Lambda(\lambda)$ is proportional to the intensity of waves proceeding parallel to the line with direction cosines λ, ν. (We call this the direction λ.) Our goal is the calculation of $\Lambda(\lambda)$, the scattered intensity spectrum, in terms of the statistical properties of the free surface z(x). In general, Λ will consist of both a discrete and a continuous portion. The discrete portion, where Λ is singular, represents plane scattered waves of finite amplitude (such as the specularly reflected wave). For such plane waves, the integral of Λ in the immediate vicinity of its singularity is equal to the average square wave amplitude.

DETERMINATION OF THE GENERALIZED SPECTRUM

We seek to represent the generalized spectrum as a power series in σ, the r.m.s. surface height:

$$G(\lambda) = \sum_{0}^{\infty}\sigma^m A_m(\lambda) \tag{6}$$

and obtain

$$e^{-i\alpha\xi}\frac{(-i\gamma\xi)^m}{m!} + \sum_{n=0}^{m}\int e^{-i\lambda\xi}\frac{(i\nu\xi)^n}{n!}\,dA_{m-n}(\lambda) = 0 \tag{7}$$

by substituting Eq. (6) into Eq. (3) and equating to zero the net coefficient of each power of σ. Eqs. (7) constitute an infinite set of simultaneous linear equations for the determination of the $A_m(\lambda)$. The details of the solution of this set are left for the appendix. From the set A_m we can construct $G(\lambda)$, and the result is

$$
\begin{aligned}
G(\lambda) = & -W_\lambda\left\{e^{-\iota\alpha\xi-\iota\nu\sigma\xi}\right\} \\
& + W_\lambda\left\{\int e^{-\iota\lambda_1\xi}\,dW_{\lambda_1}\left\{e^{-\iota\alpha\xi_1-\iota\nu\sigma\xi_1}\left(e^{\iota\nu_1\sigma\xi}-1\right)\right\}\right\} \\
& - W_\lambda\left\{\int e^{-\iota\lambda_1\xi}\,dW_{\lambda_1}\left\{\int e^{-\iota\lambda_2\xi_1}\,dW_{\lambda_2}\left\{e^{-\iota\alpha\xi_2-\iota\nu\sigma\xi_2}\cdot\right.\right.\right. \\
& \qquad \left.\left.\left.\cdot\left(e^{\iota\nu_1\sigma\xi}-1\right)\left(e^{\iota\nu_2\sigma\xi_1}-1\right)\right\}\right\}\right\} \\
& + W_\lambda\left\{\int e^{-\iota\lambda_1\xi}\,dW_{\lambda_1}\left\{\int e^{-\iota\lambda_2\xi_1}\,dW_{\lambda_2}\left\{\int e^{-\iota\lambda_3\xi_2}\cdot\right.\right.\right. \\
& \qquad \left.\left.\left.\cdot\,dW_{\lambda_3}\left\{e^{-\iota\alpha\xi_3-\iota\nu\sigma\xi_3}\left(e^{\iota\nu_1\sigma\xi}-1\right)\left(e^{\iota\nu_2\sigma\xi_1}-1\right)\left(e^{\iota\nu_3\sigma\xi_2}-1\right)\right\}\right\}\right\}\right\} \\
& - \cdots
\end{aligned}
\tag{8}
$$

Eq. (8) is readily understood as a sequence of terms in which the symbolic operator

$$X = \int e^{-i\lambda\xi}\left(e^{i\nu\sigma\xi} - 1\right) dw_{\lambda} \{ \quad \} \tag{9}$$

acts repeatedly, the number of iterations increasing uniformly through the sequence. Thus, interpreting X^{p} as a p-fold iteration of the basic operation, Eq. (9), and setting

$$f(\xi) = \int e^{-i\lambda\xi} dG(\lambda) \tag{10}$$

we can write simply

$$f(\xi) = \frac{-e^{-i\alpha\xi - i\nu\sigma\xi}}{1 + X} \tag{11}$$

Equation (11) is the natural point of departure for extension of the theory to more general situations. For mixed boundary conditions, as with electromagnetic or elastic transverse waves, it is found that the operator X must be modified by including a kernel which is essentially the Fresnel reflection (or transmission) coefficient for a plane surface. The formulae are readily adapted to three dimensional surfaces by requirements of symmetry, and to other types of incident waves by Fourier synthesis.

THE CORRELATION FUNCTION AND SCATTERED SPECTRA

We are now in a position to write down an expression for the correlation function, which by definition is

$$\varphi(\xi-\xi') = \langle f(\xi) f^{*}(\xi') \rangle \tag{12}$$

Here f^* is the complex conjugate of f and the average is to be taken over the family of surfaces ξ. We have explicitly

$$\varphi(\xi-\xi') = \left\langle \frac{e^{-i\omega(\xi-\xi') - i\partial\sigma(\xi-\xi')}}{(1+X)(1+X'^{*})} \right\rangle \tag{13}$$

in which the primes indicate that ξ is to be replaced by ξ'. Since the operator involves quantities to be averaged, it is necessary to cast Eq. (13) into a form free from operators before contemplating numerical work. A powerful procedure consists in considering the various power series representations of $\frac{1}{1+X}$ which are derivable on the principle of analytic continuation. In this way, successive approximations involving simple iterations suitable for high speed computations can be constructed as appropriate to the particular problem at hand.

APPLICATIONS

Two applications are of some current interest, as they appear to require a rather detailed theory. In scatter communications, second order effects are of interest which give rise to interactions and consequent signal distortion.

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In underwater acoustics, reverberations caused by back scattering from the boundaries of the water have long been of central importance. Figure 2 indicates a typical back scattering coefficient characteristic of ocean bottoms. The insensitivity of back scattering to frequency is striking. Since the radiation wave lengths involved here range from many times greater to many times smaller than the scale of natural objects or irregularities to be expected on sea beds, it is clear that a detailed theory is necessary for an adequate account of the situation. Whether a theory of scattering by irregular surfaces is sufficient has not yet been established.

H. W. MARSH, JR.

ACOUSTIC BACK SCATTERING FROM OCEAN BOTTOMS

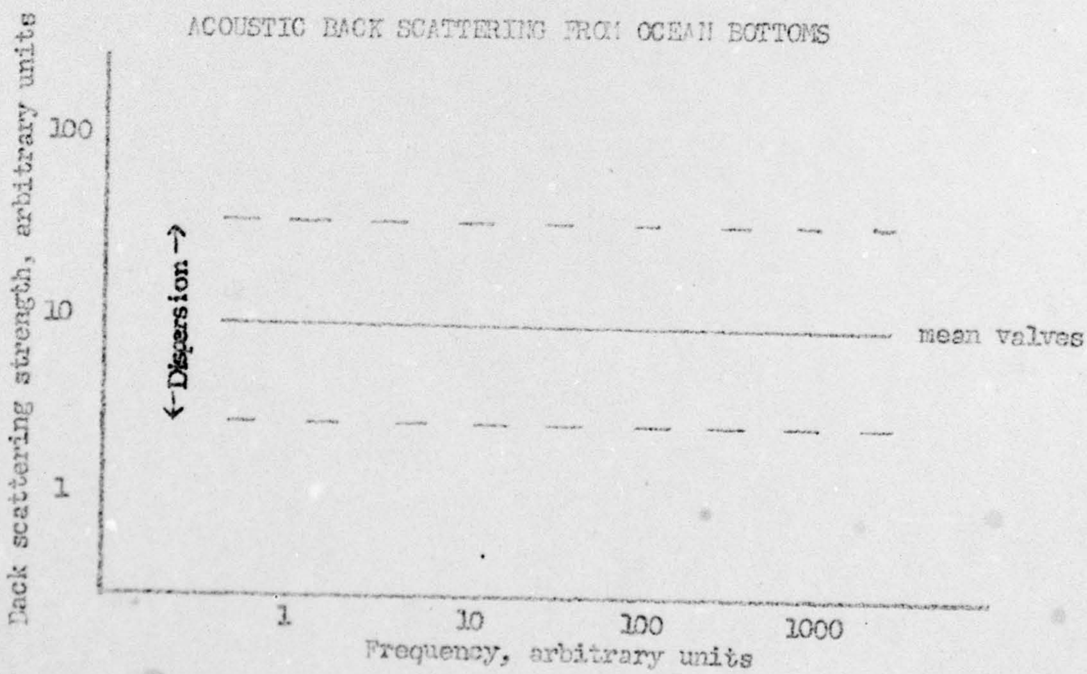

Fig. 2

APPENDIX

The fundamental equation is (Eq. 7)

$$e^{-i\alpha\xi}\frac{(-i\gamma\zeta)^m}{m!} + \sum_{n=0}^{m}\int e^{-i\lambda\xi}\frac{(i\nu\zeta)^n}{n!}\,dA_{m-n}(\lambda) = 0$$

$$\alpha^2+\gamma^2 = \lambda^2+\nu^2 = 1 \quad ; \quad \zeta = \zeta(\xi) \tag{1}$$

From (1) we can write

$$m=0 \qquad A_0(\lambda) = -W_\lambda\left\{e^{-i\alpha\xi}\right\}$$

$$m>0 \qquad A_m(\lambda) = -W_\lambda\left\{e^{-i\alpha\xi}\frac{(-i\gamma\zeta)^m}{m!}\right\} - \sum_{n=1}^{m} W_\lambda\left\{\int e^{-i\lambda\xi}\frac{(i\nu\zeta)^n}{n!}\,dA_{m-n}(\lambda)\right\} \tag{2}$$

Now substitute the l.h.s. of the second of Eqs. (2) into the r.h.s., and get

$$
\begin{aligned}
A_m = & -B_m(\lambda) + \sum_{n=1}^{m} W_\lambda \left\{ \int e^{-i\lambda_1 \xi} \frac{(i\nu_1 \xi)^n}{n!} \, dB_{m-n}(\lambda_1) \right\} \\
& + \sum_{n=1}^{m-1} W_\lambda \left\{ \int e^{-i\lambda_1 \xi} \frac{(i\nu_1 \xi)^n}{n!} \, d \sum_{p=1}^{m-n} W_{\lambda_1} \left\{ \int e^{-i\lambda_2 \xi_1} \cdot \right. \right. \\
& \qquad \left. \left. \cdot \frac{(i\nu_2 \xi_1)^p}{p!} \, dA_{m-n-p}(\lambda_2) \right\} \right\}
\end{aligned}
\tag{3}
$$

where $$B_m(\lambda) = W_\lambda \left\{ e^{-i\lambda\xi} \frac{(-i\nu\xi)^m}{m!} \right\}$$

$$\lambda_\ell^2 + \nu_\ell^2 = 1 \quad ; \quad \xi_\ell = \xi(\zeta_\ell)$$

Substitute the l.h.s. of Eq. (3) into the r.h.s., and get

$$
\begin{aligned}
A_m = & -B_m(\lambda) + \sum_{n=1}^{m} W_\lambda \left\{ \int e^{-i\lambda_1 \xi} \frac{(i\nu_1 \xi)^n}{n!} \, dB_{m-n}(\lambda_1) \right\} \\
& - \sum_{n=1}^{m-1} W_\lambda \left\{ e^{-i\lambda_1 \xi} \frac{(i\nu_1 \xi)^n}{n!} \, d \sum_{p=1}^{m-n} W_{\lambda_1} \left\{ \int e^{-i\lambda_2 \xi_1} \frac{(i\nu_2 \xi_1)^p}{p!} \, dB_{m-n-p}(\lambda_2) \right\} \right\} \\
& - \sum_{n=1}^{m-2} W_\lambda \left\{ e^{-i\lambda_1 \xi} \frac{(i\nu_1 \xi)^n}{n!} \, d \sum_{p=1}^{m-n-1} W_{\lambda_1} \left\{ \int e^{-i\lambda_2 \xi_1} \cdot \right. \right. \\
& \qquad \left. \left. \cdot \frac{(i\nu_2 \xi_1)^p}{p!} \, d \sum_{q=1}^{m-n-p} W_{\lambda_2} \left\{ \int e^{-i\lambda_3 \xi_2} \frac{(i\nu_3 \xi_2)^q}{q!} \, dA_{m-n-p-q}(\lambda_3) \right\} \right\} \right\}
\end{aligned}
\tag{4}
$$

From Eq. (4) the result of repeated substitutions is clear. We can now form $\sum \sigma^m A_m(\lambda)$, which is

$$
\begin{aligned}
G(\lambda) = \; & -W_\lambda\left\{e^{-\iota\alpha\xi-\iota\nu\sigma\xi}\right\} \\
& + W_\lambda\left\{\int e^{-\iota\lambda_1\xi} dw_{\lambda_1}\left\{e^{-\iota\alpha\xi_1-\iota\nu\sigma\xi_1}\left(e^{\iota\nu_1\sigma\xi}-1\right)\right\}\right\} \\
& - W_\lambda\left\{\int e^{-\iota\lambda_1\xi} dw_{\lambda_1}\left\{\int e^{-\iota\lambda_2\xi_1} dw_{\lambda_2}\left\{e^{-\iota\alpha\xi_2-\iota\nu\sigma\xi_2}\cdot\right.\right.\right. \\
& \qquad \left.\left.\left.\cdot\left(e^{\iota\nu_1\sigma\xi}-1\right)\left(e^{\iota\nu_2\sigma\xi}-1\right)\right\}\right\}\right\} \\
& + W_\lambda\left\{\int e^{-\iota\lambda_1\xi} dw_{\lambda_1}\left\{\int e^{-\iota\lambda_2\xi_1} dw_{\lambda_2}\left\{\int e^{-\iota\lambda_3\xi_2}\cdot\right.\right.\right. \\
& \qquad \left.\left.\left.\cdot dw_{\lambda_3}\left\{e^{-\iota\alpha\xi_3-\iota\nu\sigma\xi_3}\left(e^{\iota\nu_1\sigma\xi}-1\right)\left(e^{\iota\nu_2\sigma\xi_1}-1\right)\left(e^{\iota\nu_3\sigma\xi_2}-1\right)\right\}\right\}\right\}\right\} \\
& - \cdots
\end{aligned}
\tag{5}
$$

This can be abbreviated by writing

$$H_n = e^{-i\lambda_{n+1}\xi_n}\left(e^{i\nu_{n+1}\sigma\xi_n} - 1\right)$$

$$\lambda_0 = \lambda, \quad \nu_0 = \nu \quad ; \quad \xi_0 = \xi$$

$$K_n = W_{\lambda_n}\left\{e^{-i\alpha\xi_n - i\nu\sigma\xi_n}\right\}$$

$$F(\xi) = \int e^{-i\lambda\xi}\, dW_\lambda$$

Then

$$F(\xi) = -e^{-i\alpha\xi - i\nu\sigma\xi} + \int H_0\, dK_1$$

$$- \int H_0\, dW_{\lambda_1} \int H_1\, dK_2 + \int H_0\, dW_{\lambda_1} \int H_1\, dW_{\lambda_2} \int H_2\, dK_3$$

$$- \cdots$$

Further, interpreting the iterated integrals as the result of an operator (say X) acting repeatedly, we can write simply

$$F(\xi) = - \frac{e^{-i\alpha\xi - i\nu\sigma\xi}}{1 + X} \qquad (6)$$

X being the operator $\int H\, dW\{\ \}$

It will be noticed that the operator X consists of two terms, arising from the two additive parts of H. The second of these, i.e., that associated with -1, is clearly the identity operator (with change of sign). Hence, we can write

$$L_n = e^{-i\lambda_{n+1}\xi_n + i\nu_{n+1}\sigma\xi_n}$$

and

$$F(\xi) = \frac{-e^{-i\alpha\xi - i\nu\sigma\xi}}{Y} \qquad (7)$$

Y being the operator $\int L\,dW\{\ \}$. Equation (7) is simply the operational solution to equation (1), and as such provides a complete check on the accuracy of our results.

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